Journal of Engineering Physics and Thermophysics, Vol. 78, No. 3, 2005

RADIATION TRANSFER IN TECHNOLOGICAL PROCESSES

CALCULATION OF THE CHARACTERISTICS OF TRANSFER OF THERMAL RADIATION IN THE WORKSPACE OF AN ANNULAR FURNACE

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UDC 536.33

A study has been made of the distinctive features of radiation transfer in the workspace of an annular furnace and the accuracy of calculation of radiation fluxes incident on the furnace walls in operation of the annular furnace and on the surface of cylindrical and rectangular metallic billets worked. The possibility of reducing the dimension of the problem on calculation of thermal-radiation fluxes from a three-dimensional problem to a two-dimensional one has been substantiated. The influence of the selectivity of the radiation of flue gases and the emissivity factor of refractory furnace surfaces on the integral radiation-flux densities has been considered.

Annular furnaces with a moving hearth are widely used for heating of billets worked on rolling mills [1]. In these furnaces, billets lying immovably on the rotary hearth traverse, together with it, the continuous, welding, and soaking zones. A billet must be heated to the necessary temperature in one complete rotation of the hearth. The billets are charged and discharged using special machines. The annular hearth of the furnace moves by jerks, rotating by an angle corresponding to the distance between two neighboring billets in each jerk. The rotational velocity of the hearth may change with the size of the billet heated. Special sand or water seals are manufactured to avoid the suction of cold air into the workspace between the furnace walls and the rotary hearth. The general view of an annular furnace is presented in Fig. 1.

Annular furnaces may burn both gaseous and liquid fuels. Burners are installed on both the exterior and interior furnace walls. Arch heating of annular-hearth furnaces is finding increasing use. The combustion products are extracted at one or more sites. The arrangement of burners and fume offtakes is one of the most important characteristics of annular-hearth furnaces, depending on which a furnace may operate according to the continuous or chamber regime. One uses flame burners when the arrangement is lateral, plane-flame burners in the case of arch heating, and low-pressure atomizers in fuel-oil heating. Annular-hearth furnaces are usually equipped with recuperators. The most important advantages of annular furnaces concern the possibility of diminishing substantially the overall dimensions of a furnace and of utilizing thermal energy more efficiently.

The dimensions of furnaces depend on both their output and the form and size of the billets heated. The output of furnaces reaches 70 tons/h in individual cases; their diameter along the axial line of the hearth is 25–30 m and their width is 6 m or more. The height of a furnace is selected based on the optimum distance between the burners and the heated metal and the furnace arch. To avoid fusion of the metal lateral burners must be above it at a distance no smaller than 450–500 mm and approximately at the same distance from the arch. Billets are spaced 100–200 mm apart in the furnace. In the case of multirow stacking, the distance from the ends of the billets to the furnace walls must be about 0.5 m.

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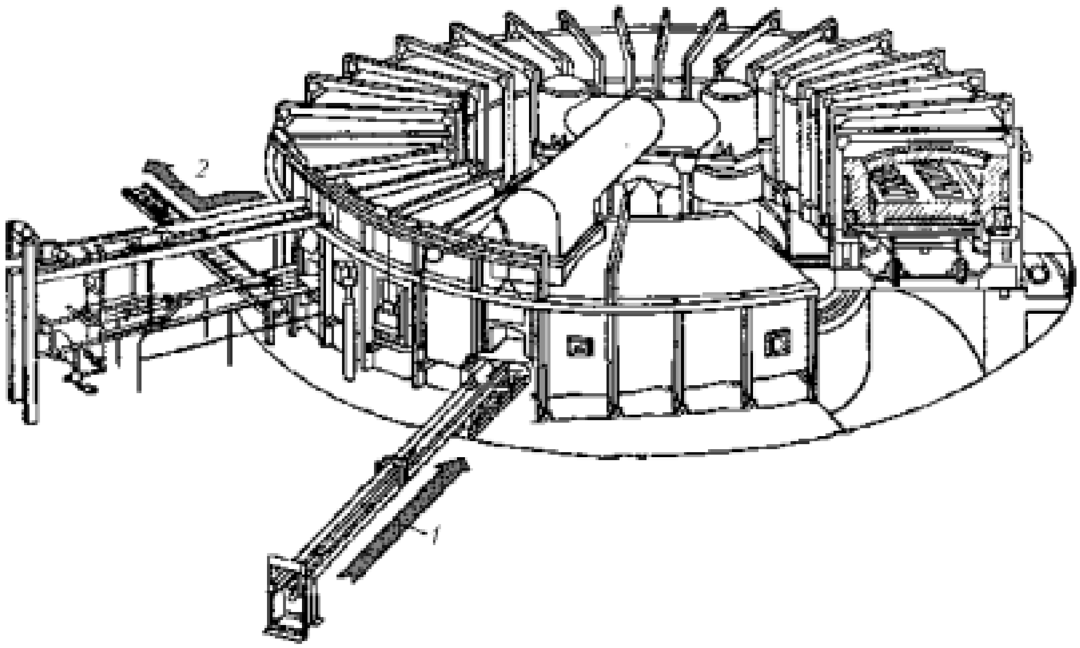

Fig. 1. General view of the furnace with a rotary hearth: 1) charging of a steel billet; 2) discharging.

Rolling mills with a very wide assortment of metal require furnace units possessing a high heat-engineering flexibility and adapted to transition from the chamber operating regime to a continuous regime and conversely. An annular-hearth furnace is exactly such a unit. Owing to the presence of burners uniformly arranged on the furnace's circle, one can distribute the fuel supply in accordance with the requirements of the temperature regime: uniformly in the chamber regime and nonuniformly in the continuous regime. Burners ensuring nearly 75–80% of the furnace's heat power are placed on the external circle, and those ensuring 20–25% are placed on the internal circle. There can be another fuel distribution up to the complete switching-off of the burners in the warmup zone. Such an operating regime of the furnace requires burners with control limits as wide as possible.

In furnaces with an annular hearth, its surface is far from being totally occupied by metal; therefore, the brickwork of the hearth plays a fairly active part in radiation heat exchange and contributes to a more uniform heating of the metal. Billets stacked with gaps and free portions of the hearth radiate and reflect heat rays, which partially arrive at the lower surface of the billets and contribute to their more uniform nearly symmetric heating. As is well known, the coefficient of asymmetry of heating is $\mu = q_1/(q_1 + q_2)$, where q_1 and q_2 are the heat fluxes onto the heated billet from above and from below respectively. Symmetric heating occurs for $\mu = 0.5$. It has been established that in annular-hearth furnaces, we have $\mu = 0.55$–0.58 in heating of a round billet. This shows that, although heating is not totally symmetric, round billets receive a fairly large quantity of heat from the brickwork at the bottom. We should note that the hearth brickwork particularly intensely radiates heat to billets immediately after their charging, when it has just left the high-temperature zone and the billets are cold. At the beginning of the warmup zone, the brickwork cools down slightly but then its temperature begins to increase with warmup of a billet. In addition to radiation, the heat is transferred to the billets by conduction, too.

The above-described technological features of operation of an annular furnace demonstrate the complex character of heat exchange in its workspace. Such furnaces are very energy-consuming, due to their large overall dimensions and high output. Therefore, it is topical to construct a mathematical model describing the processes of heat exchange in an annular furnace. Such a model is necessary for optimization of thermal operating regimes of the furnace, which enables one to minimize energy consumption. The characteristic operating temperatures of an annular furnace are in the range 1000–1200°C. At such temperatures, the contribution of radiation energy transfer to the total energy exchange may attain 90% or more (see, for example, [2]). For this reason, the accuracy of the models mentioned above will primarily depend on the accuracy of calculation of the radiation heat exchange. Therefore, correct calculation of the characteristics of thermal radiation in the workspace of annular furnaces is a key problem whose solution will enable one to develop operating regimes of annular furnaces that are optimum in energy indices. The complexity of the problem is associated with the necessity of solving a system of integro-differential equations on a vast number of nodes in a space with a complex three-dimensional geometry. The large number of nodes in this case is determined by the large overall dimensions of the furnace and the necessity for highly accurate calculations. The number of nodes covering the entire workspace of the annular furnace and necessary for calculation may attain several million. Calculation of a large number of variants on long time intervals will be required for finding the optimum operating regimes. Such a path is inefficient. It is significant that calculation of the characteristics of radiation transfer takes most of the computational time, since in this case one must solve the integro-differential equation for a wide spectrum of radiation wavelengths. One possible way of solving the problem is reduction in the dimension of the problem from a three-dimensional problem to a two-dimensional one. This work seeks to investigate such a possibility and the correctness of reduction in the dimension of the problem of calculation of the transfer of thermal radiation in the space of an annular furnace.

Mathematical Model of Radiation Transfer in the Workspace of an Annular Furnace. In calculating the characteristics of radiation transfer in selectively radiating, absorbing, and scattering media, we must allow for multiple processes of reradiation and scattering of radiation on solid-phase particles, the selectivity of radiation of the furnace medium, and a complex configuration of a radiating volume. The characteristics of radiant heat exchange are traditionally found by solution of the equation of radiation transfer [3, 4]. On condition of a local thermodynamic equilibrium, this equation expresses the law of conservation of radiant energy in its propagation in the absorbing, radiating, and scattering medium and has the following form:

$$\mathbf{l}\cdot\nabla I_\lambda(\mathbf{r},\mathbf{l}) + [\chi_\lambda(\mathbf{r}) + \sigma_\lambda(\mathbf{r})]\, I_\lambda(\mathbf{r},\mathbf{l}) = \chi_\lambda(\mathbf{r})\, B_\lambda(T(\mathbf{r})) + \frac{\sigma_\lambda(\mathbf{r})}{4\pi}\int_{4\pi} p_\lambda(\mathbf{r},\mathbf{l},\mathbf{l}')\, I_\lambda(\mathbf{r},\mathbf{l}')\, d\Omega' . \quad (1)$$

Boundary conditions to Eq. (1) are determined by the processes of radiation and reflection on boundary surfaces and may be written in the general case as [5]

$$I_\lambda(\mathrm{P},\mathbf{l})\big|_{(\mathbf{l}\cdot\mathbf{n})<0} = I_{0\lambda}(\mathrm{P},\mathbf{l}) + \frac{1}{\pi}\int_{2\pi} \rho_\lambda(\mathrm{P},\mathbf{l},\mathbf{l}')\cdot I_\lambda(\mathrm{P},\mathbf{l}')\cdot(\mathbf{l}'\cdot\mathbf{n})\, d\Omega' . \quad (2)$$

Based on the radiation-intensity field calculated from Eqs. (1) and (2), we determine two more energy quantities necessary for subsequent computation of the medium's temperature: the bulk density of radiation heat sources/sinks at each point of the medium:

$$\mathrm{div}\,\mathbf{Q}_\mathrm{r} = \int_0^\infty \chi_\lambda(\mathbf{r}) \left(4\pi B_\lambda(T(\mathbf{r})) - \int_{4\pi} I_\lambda(\mathbf{r},\mathbf{l})\, d\Omega \right) d\lambda , \quad (3)$$

the local densities of the resulting radiation flux onto heat-absorbing surfaces if they are present in the system:

$$q_\mathrm{w}^\mathrm{res}(\mathrm{P}) = \int_0^\infty \varepsilon \left(\int_{2\pi} I_\lambda(\mathrm{P},\mathbf{l})\cdot(\mathbf{l}\cdot\mathbf{n})\, d\Omega - \pi B_\lambda(T_\mathrm{w}(\mathrm{P})) \right) d\lambda . \quad (4)$$

Optical Properties of Flue Gases. The optical characteristics of flue gases are an important characteristic necessary for correct calculation of radiation transfer inside the annular furnace. We briefly dwell on this problem.

Molecular gases (CO, CO_2, H_2O, SO_2, and others) which are optically active in the infrared spectrum enter into the composition of the combustion products of natural gas. It takes much computer time to calculate the emittance of a mixture of these gases by the "line-after-line" method [5], which is completely unsuitable for engineering and diagnostic calculations of furnace chambers. The difficulties of such a calculation are associated with the necessity of selecting very small spectral intervals (10^{-4}–10^{-2} cm^{-1}). In this connection, in calculations, one selects a spectral interval containing several lines and thereafter describes the spectroscopic properties of gases in it based on model representations.

The most widespread are the Elsasser and Goody models and different combinations of them. An infinite set of equidistant lines of the same intensity is taken in the Elsasser model, whereas the statistical Goody model assumes a random distribution of the positions and intensity of spectral lines. In [6], the above models of bands are tested for accuracy in the spectral range 150–8000 cm^{-1} and the temperature interval 300–1500 K; check calculation of the spectral properties of carbon dioxide and water vapor by the "line-after-line" method is carried out and a comparison to the results of calculation based on model representations is made. The comparison has shown that the statistical Goody model yields the best approximation for a homogeneous layer and physical conditions characteristic of furnace chambers.

In the case of a nonuniform distribution of molecular gases by density and temperature the problem of calculation of their emittance is largely complicated. A method proposed by Curtis [7] and Godson [8] independently is widely used for these purposes at present. Its essence is that the transmission along the inhomogeneous path is replaced by that along a hypothetical homogeneous layer. The constants determining the transmission of a hypothetical layer are selected from the conditions of coincidence of the above transmissions in the approximations of strong and weak lines [9]. Numerous results obtained using the Curtis–Godson method and their comparison to experimental data and results of the "line-after-line" method [6] show that this approximation is quite satisfactory for the Lorentz contour of spectral lines in the absence of strong inhomogeneities in the radiating volume.

A procedure of calculation for strongly inhomogeneous volumes of molecular gases for a chaotic set of Lorentz spectral lines of the same intensity has been proposed in [10]. As has been shown in [6], this method yields satisfactory results.

In [11–13], computational formulas for determination of the effective spectral-absorption coefficient of molecular gases in a finite spectral interval with allowance for scattering are given, and the results obtained from the formulas derived are compared to the data of other authors and to experiment. The dependences obtained are based on the Elsasser model of an absorption band, a detailed integration of the radiation flux along the contour of the absorption line, and mathematical processing of the results. The final formulas proposed enable one to allow for the fine structure of the absorption line with a high degree of accuracy in calculating the effective absorption coefficient for scattering and nonscattering media and substantially (by several orders of magnitude) reduce the time of calculation or processing of the signal, which is of particular importance for remote diagnostics and control of rapid processes. At the same time, the application of them to calculations in a wide spectrum (1–6 μm) is also quite a resource-intensive process.

As has already been noted, the above methods of calculation of the emittances of a mixture of molecular gases are inefficient for problems of modeling and diagnostics of furnace chambers with the aim of determining the optimum regime of their operation, where one important index is the calculation time. The procedure described in [13] is more appropriate; according to it, the spectral-absorption coefficient of the gas phase of a furnace is calculated from the formula

$$\chi (\lambda, T) = \frac{0.3}{t^2} \sum_{i=1}^{N_g} p_i \exp \left[A_i (\lambda) + \frac{B_i (\lambda)}{t} + \frac{C_i (\lambda)}{t^2} \right], \tag{5}$$

where $t = 0.001\ T$, p_i is the partial pressure of the ith gas in the mixture, and $A_i(\lambda)$, $B_i(\lambda)$, and $C_i(\lambda)$ are the coefficients selected empirically. This dependence, obtained at the Institute of Physics of the National Academy of Sciences of Belarus based on an analysis of numerous experimental data [14, 15], is applicable in the temperature range 300–3000 K; the approximation errors do not exceed 10%. The values of the coefficients A, B, and C for the CO, CO_2, and H_2O gases which make the main contribution to the radiation of the gas phase of the furnace medium are given in [13]. Calculation from formula (5) requires insignificant consumption of computer time. Numerous applications of the above procedure to calculation of the optical properties of the gas mixture and the characteristics of radiation transfer in furnace chambers [16, 17] have shown their high efficiency. In this connection, in what follows in this work, the absorption coefficients of the gas mixture will be calculated from the procedure described.

When (5) is used, it should be borne in mind that in the case of the equality of all three coefficients to zero the absorption coefficient is also equal to zero. Also, it is noteworthy that the absorption indices calculated on neigh-

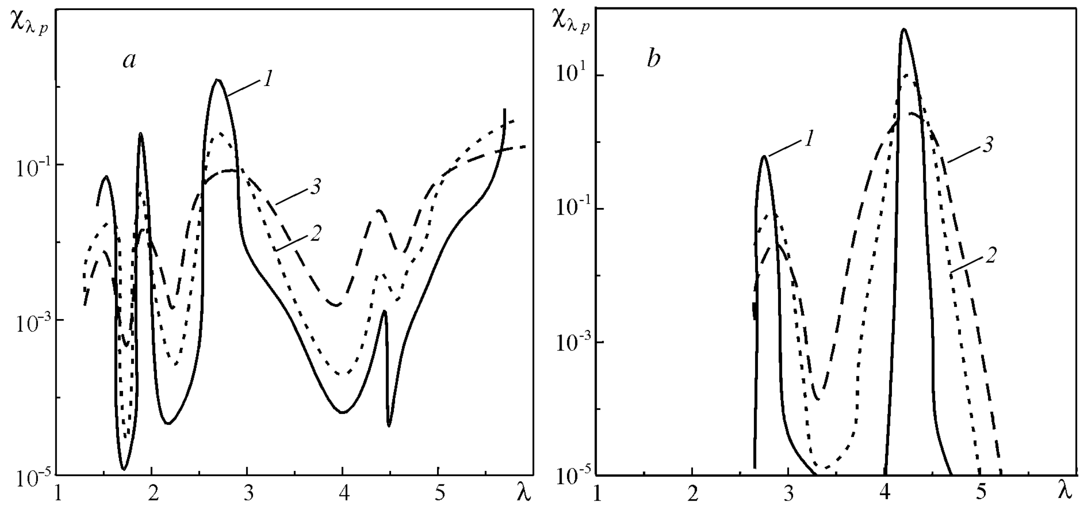

Fig. 2. Specific spectral absorption coefficient of an H_2O vapor (a) and a CO_2 gas (b) vs. infrared-radiation wavelength: 1) T = 300; 2) 1000; 3) 1700 K. $\chi_{\lambda p}$, $(\text{cm}\cdot\text{atm})^{-1}$; λ, μm.

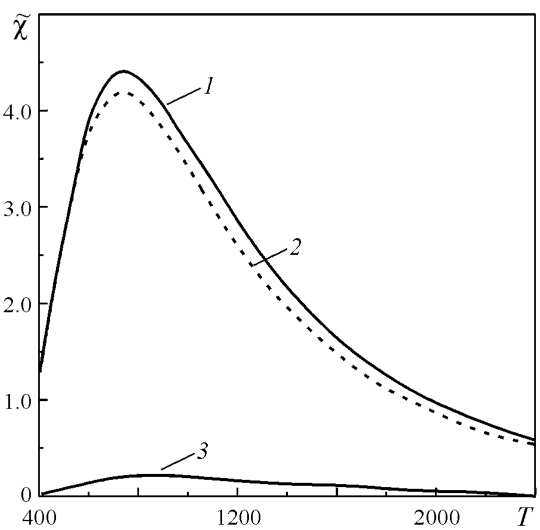

Fig. 3. Mean-integral absorption coefficients of H_2O and CO_2 gases and their mixture vs. temperature: 1) $H_2O + CO_2$; 2) CO_2; 3) H_2O. $\tilde{\chi}$, m^{-1}; T, K.

boring tabulated values of the wavelength and not the coefficients themselves in (5) should be interpolated for a prescribed radiation wavelength λ. Figure 2 gives as an example results of calculation of the specific (per partial-pressure atmosphere) absorption coefficient of the H_2O and CO_2 gases. It can be seen that this dependence is very complex in character.

The absorption coefficient (mean integral over the spectrum) of the furnace medium, or the "gray" coefficient, is determined by the following formula:

$$\tilde{\chi} = \frac{\pi \int_0^\infty \chi_\lambda B_\lambda(T)\, d\lambda}{\sigma_0 T^4} . \tag{6}$$

The temperature dependence of the "gray" absorption coefficient of the furnace medium for the average composition of flue gases (partial pressure p_{H_2O} = 0.11 and p_{CO_2} = 0.13) is given in Fig. 3; it is seen that for the range of annular-furnace temperatures 800–1300^{o}C of interest, the "gray" absorption coefficient of the furnace medium is more than unity. Consequently, with allowance for the geometry of the furnace (arch height 1.5 m and hearth width 3.5 m), the minimum optical thickness of its internal space exceeds 1.5, whereas the relation of the optical thicknesses over the furnace width and height is $\delta = \tau_Z/\tau_X = 2.3$.

In [18], we reviewed modern methods of solution of the equation of radiation transfer. As has already been noted, it takes much computer time to solve this equation in the annular-furnace space with an accuracy acceptable for practice. An efficient method of solution of this problem is reduction in the dimension of the problem from 3*D* to 2*D*. Such a technique is allowable for practical applications if the arising errors do not exceed 10%; therefore, in what follows we investigate the errors arising with reduction in the dimension of the problem on calculation of radiation energy transfer in a radiating, scattering, and absorbing medium.

Influence of the Dimension of the Radiation-Transfer Model on the Accuracy of Determination of the Characteristics of Radiation Heat Exchange. In many practical problems, calculation of the radiation fluxes along the direction singled out in space involves certain computational and technical difficulties: first, this is associated with the difficulty of software realization of a numerical algorithm of solution of the multidimensional transfer equation; second, it takes much computer time to calculate the characteristics of radiation in this case; therefore, in actual practice, one usually uses simplified models of smaller dimension. The main criterion of the possibility or impossibility of reducing the dimension of the problem is the error of the final result which is introduced by such a reduction. Data on this problem available in the scientific literature are very scanty.

First we analyze errors arising with reduction in the dimension of the problem of calculation of radiation transfer. We investigate the errors of calculation of the basic energy quantities that appear in the energy-balance equation and that are the most important in determining the temperature state of the medium. The first of them is the density of the resulting-radiation flux at the boundary of the radiating medium:

$$Q_{\text{res}}(\mathbf{r}) = \int_{4\pi} I(\mathbf{r}, \mathbf{l})(\mathbf{l}\cdot\mathbf{n})\, d\Omega ; \tag{7}$$

the second quantity is the angular mean intensity of radiation emerging from the medium:

$$J = \frac{1}{4\pi} \int_{4\pi} I(\mathbf{r}, \mathbf{l})\, d\Omega . \tag{8}$$

The dependence of the error of calculation of the mean intensity and the radiation flux in the case of replacement of the three-dimensional model by a two-dimensional one on the optical parameters of the medium and its geometry was investigated with the example of the problem on radiation transfer inside a domain having the shape of a parallelepiped with sides L_X, $L_Y = L_X$, and L_Z and optical thicknesses equal to τ_X, $\tau_Y = \tau_X$, and τ_Z respectively. It was assumed in the calculations that the domain is filled with the selectively radiating, absorbing, and scattering medium. The optical thickness of the medium was determined as $\tau = \bar{\chi}L$, where L is the geometric dimension along the axis in question. We calculated the mean intensity and the density of the radiation flux in its middle cross section having the shape of a square and an optical thickness of τ_X. The optical thickness was varied over the parallelepiped height τ_Z. We investigated the influence of the ratio $\delta = \tau_Z/\tau_X$ on the error of calculation of the values of the flux and mean intensity of radiation at a point located on the median of the middle cross section of the parallelepiped. The error of calculation of the radiation flux and mean intensity was analyzed based on the following relations:

$$\varepsilon(Q_{\text{res}}) = \left| \frac{Q_{\text{res}}(D) - Q_{\text{res}}(D-1)}{Q_{\text{res}}(D-1)} \right| , \tag{9}$$

$$\varepsilon(J) = \left| \frac{J(D) - J(D-1)}{J(D-1)} \right| , \tag{10}$$

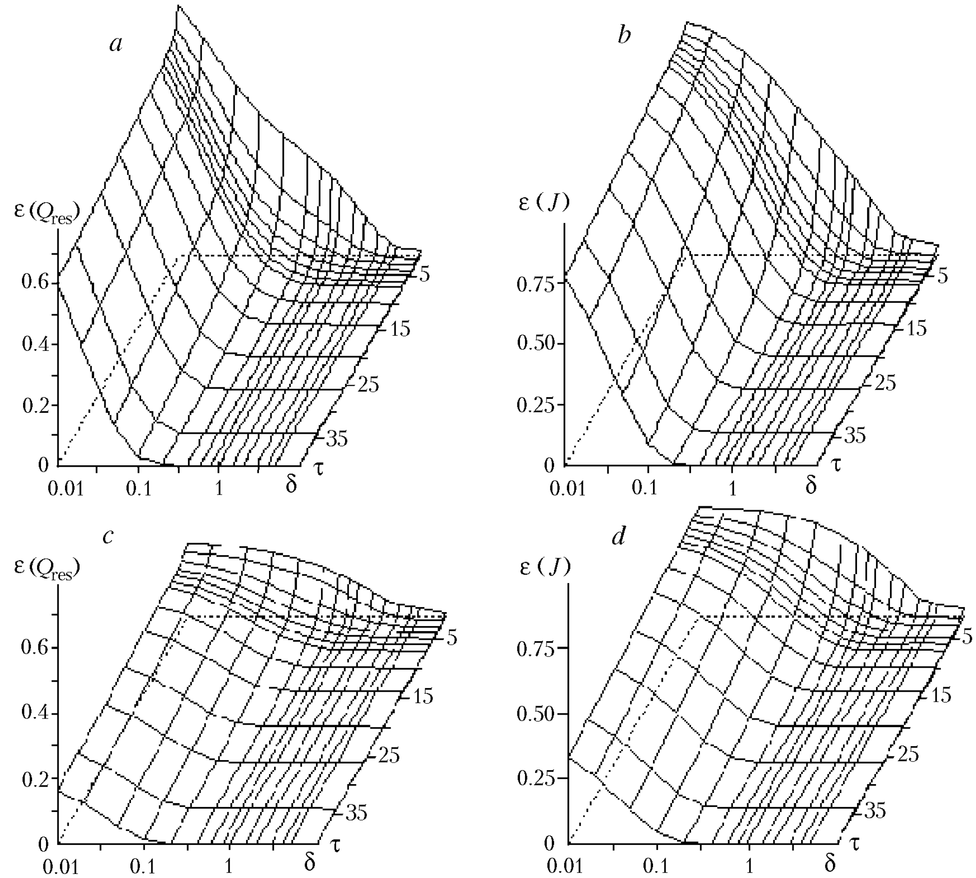

Fig. 4. Distribution of the computational errors for the value of the incident radiation flux (a and c) and the mean intensity in the medium's volume (b and d) with replacement of the 3*D* problem of transfer by a 2*D* problem over the middle cross section of the parallelepiped as a function of the optical density of the medium τ and the ratio of the optical height to the optical width of the parallelepiped δ: a and b) free boundaries; there is no external radiation; c and d) diffuse boundaries; ε = 0.8.

where D is the dimension of the problem (D = 2, 3). To calculate the values of Q_{res} and J we used the method described in [18]. The investigations were carried out for the range of optical thicknesses $\tau \in [0.01 - 10]$. The number of angular subdivisions was prescribed to be equal to 101. The error of solution of the radiation-transfer equation did not exceed 0.1%. For each value of τ_X we varied τ_Z so that the parameter $\delta = \tau_Z/\tau_X$ ran through the range from 1 to 40.

The data obtained are presented in Fig. 4. Figure 4a and b gives results for the case where the boundary surfaces of the domain are transparent and there is no external radiation. The data given in Fig. 4c and d have been obtained in calculating the transfer of radiation to the domains with diffusely reflecting boundary surfaces. The emissivity factor of the boundary surface was prescribed to be ε = 0.8, and the temperature of the boundary T_w had a value such that the radiation intensity at this temperature was equal to $B(T_w) = 0.1B(T_{max})$. Here T_{max} is the maximum temperature at the center of a nonuniformly warmed-up gas domain, which was taken to be 200 K in the calculations.

An analysis of the results presented in Fig. 4 shows that for small optical thicknesses typical of the NO and CO gases and weak H_2O and CO_2 bands, the replacement of the three-dimensional model by a two-dimensional one leads to considerable errors even for strongly extended rectangular domains. Thus, for $\tau_X \le 0.1$, the error of calculation of the mean radiation intensity amounts to 13% even for δ = 40. For the radiation flux the error is much smaller, since the contribution of radiation for the direction singled out depends on the cosine of the angle of incidence and rapidly decreases when $\cos\Omega \to 0$. For the optical thicknesses $\tau_X \ge 1$ the relative errors $\varepsilon(Q_{res})$ and $\varepsilon(J)$ rapidly de-

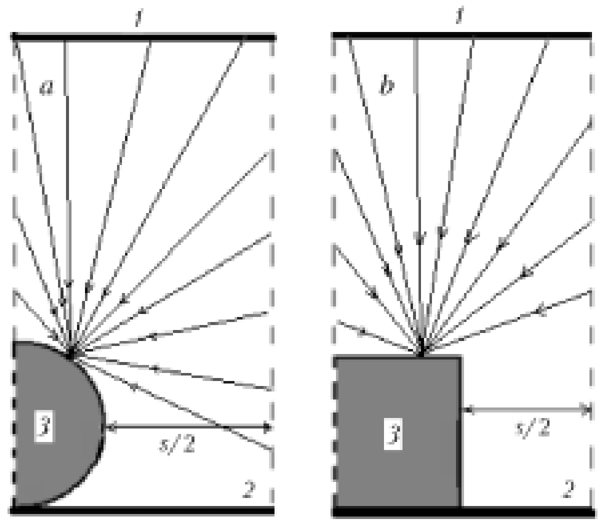

Fig. 5. Computational scheme for determination of the density of radiation fluxes on the surface of steel billets with circular (a) and square (b) cross sections: 1) arch of the annular furnace; 2) hearth of the annular furnace; 3) billet (s is the distance between the billets).

crease (several percent even for $\delta > 2$) with growth in the optical thickness and the parameter δ. For $\tau_X > 3$, which is typical of the CO_2 absorption bands, we may use the approximation of a plane layer even for $\delta \sim 1$ to compute the radiation flux.

In addition, it should be noted that in the case in question the error of solution is much higher than that in replacement of the two-dimensional problem by a one-dimensional one for the same values of the parameter δ [19]. This is particularly noticeable for small optical thickness ($\tau_X < 0.1$) for which the solution error may attain 100%. With increase in the parameter δ, the value of the error decreases much more slowly than that for the case of replacement of the two-dimensional model by a one-dimensional one. Also, we must note that in this case allowance for the processes of scattering influences the computational error only slightly.

The presence of diffusely reflecting and radiating boundaries contributes to a decrease in the solution error with replacement of the three-dimensional model by a two-dimensional one. Thus, for example, the relative error of calculation of the flux and mean intensity of radiation incident on boundary surfaces decreases approximately four and two times respectively.

Summarizing the investigation results, we should note that for an error of 10%, which may be assumed to be allowable in solving most practical problems, the replacement of the three-dimensional model by a two-dimensional one is expedient if $\tau_X \geq 0.5$ and $\delta \geq 2$. On this basis, we may infer that consideration of radiation transfer in the two-dimensional formulation is sufficient if we obtain an infinite width of the furnace hearth. In this case, the computational error for the flux does not exceed 2%. The side lining of the furnace will not be considered in determining the radiation heat fluxes onto steel billets and the computational scheme of the process of heating of steel billets and propagation of radiation in the annular furnace will be such as is shown in Fig. 5. This substantially reduces the calculation time and simplifies the mathematical model of conjugate heat exchange in the annular-furnace space in heating of steel billets of circular and square (Fig. 5b) cross sections.

Influence of the Spectral Properties of Flue Gases and Refractory Materials on the Value of the Radiation Fluxes to the Metal Surface. Based on the mathematical model developed, we created a computer program enabling one to calculate the characteristics of radiation transfer in the space of an annular furnace with allowance for the geometry of the cross section of steel billets (a circle in Fig. 5 and a square in Fig. 5b).

Figure 6 gives the densities (averaged over the billet surface) of spectral radiation fluxes onto the surface of metal $\langle Q_{res} \rangle = \oint q_{res}(\zeta) d\zeta / \oint d\zeta$ of circular (a) and square (b) cross sections (the diameter of the circle or the side of the square are equal to 0.3 m; $s = 0.1d$) at different temperatures of the billet surface and the following parameters of

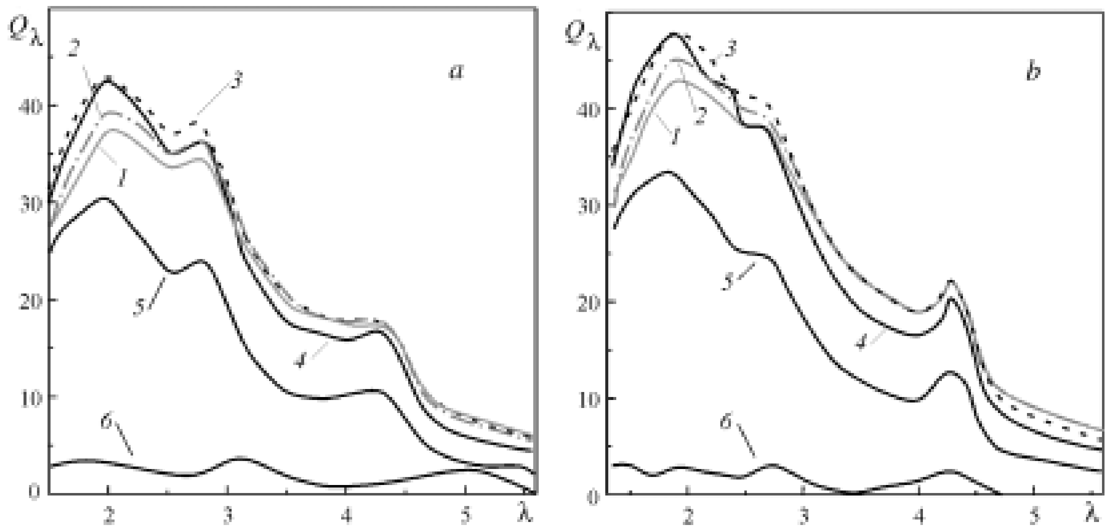

Fig. 6. Spectral density of the resulting radiation flux on the surface of billets with circular (a) and square (b) cross sections, averaged over the billet surface, at different billet temperatures: 1) 200; 2) 400; 3) 600; 4) 800; 5) 1000; 6) 1200^oC. Q_λ, kW/(m^2·μm); λ, μm.

the furnace: temperature of the flue gases $T_g = 1250^oC$, temperatures of the hearth and arch of the furnace 1200 and 1240^oC respectively, and arch height H = 1 m; the emissivity factor of the lining was selected to be 0.75; for the metal we prescribed ε = 0.5, 0.6, 0.8, and 0.9 for T_{sr} = 0, 500, 800, and 1200^oC respectively. From the above results it is clear that, as the surface temperature of the metal increases, we observe a growth in the density of the radiation flux onto the surface followed by its decrease. This is caused by the simultaneous change in the emissivity factor of the metal and its temperature. The fact is that the resulting flux (given on the plots) in the region of low temperatures of the metal surface (Fig. 6) depends on the emissivity factor more strongly than on the difference of the temperatures of the furnace medium and the metal and hence on the incident radiation flux (10), which increases with growth in the difference of the temperatures of the radiating medium and the surface:

$$Q_{res}(\zeta) = \varepsilon(\zeta)\,[Q_{inc}(\zeta) - \pi B(T_m(\zeta))]\,, \qquad (11)$$

where $Q_{inc}(\zeta, \mathbf{r}) = \int_{2\pi} I_\lambda(\zeta, \Omega)(\Omega\cdot\mathbf{n})d\Omega$ for $\Omega\cdot\mathbf{n} \ge 0$ is the density of the radiation flux incident on the metal surface.

Based on the procedure presented in this work and in [18], we developed a computer program for computation of the characteristics of radiation transfer in the workspace of an annular furnace. Using this computer program, we investigated the influence of the furnace-arch height and the emissivity factor of the lining on the maximum specific output of the furnace at a constant temperature along the furnace channel. The basic calculated parameters were as follows: 5KhNM-steel billet of a circular cross section of diameter 200 mm, relative distance between billets s/d = 0.25, billet length equal to the furnace width, and velocity of motion of the hearth 4 mm/sec; the height of the furnace arch changed from 0.4 to 1.2 m; the emissivity factor of the furnace lining changed from 0.1 to 0.9; the temperature along the furnace channel was constant and equal to 1523 K; the final temperature of heating of the billet was 20 K lower than the furnace temperature; the heating of the billet was completed if the final heating temperature attained and the maximum temperature difference over the cross section of the biller did not exceed 10 K. The maximum specific output of the furnace was determined as follows [20]:

$$U = \frac{\pi\rho_m d^2}{4\,(d+s)\,t_h}\,. \qquad (12)$$

The calculation results demonstrating the influence of the height of the furnace arch and the emissivity factor of the lining on the maximum specific output of the furnace at a constant temperature along the furnace channel are

TABLE 1. Specific Output of the Furnace as a Function of the Height of Its Arch and the Emissivity Factor of the Lining

H, m	Emissivity factor of the lining				
	0.1	0.3	0.5	0.7	0.9
0.4	0.260	0.252	0.246	0.241	0.233
0.6	0.289	0.284	0.280	0.278	0.276
0.8	0.296	0.291	0.287	0.286	0.285
1.0	0.298	0.293	0.289	0.289	0.287
1.2	0.298	0.294	0.290	0.289	0.288

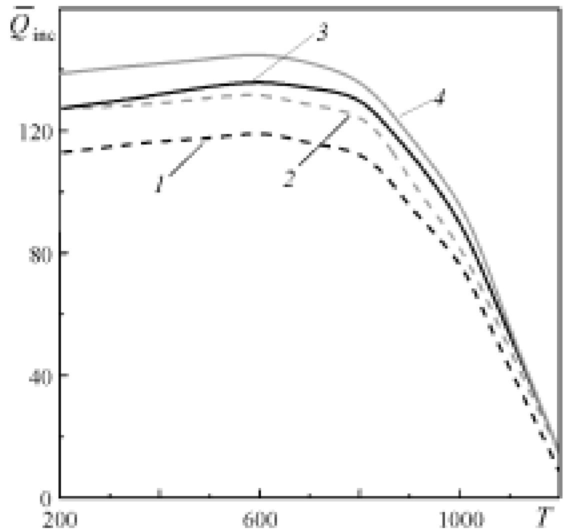

Fig. 7. Comparison of the radiation flux incident on the metal and integral over the spectrum, which is calculated with allowance for the radiation spectrum of the furnace medium (1 and 2) and with the use of the mean-integral (gray) absorption coefficient (3 and 4): 1 and 3) for a billet with a circular cross section; 2 and 4) for a billet with a square cross section. $\bar{Q}_{inc}$, kW/m^2; T, ^{o}C.

given in Table 1. It is seen that the furnace output grows with decrease in the emissivity factor of the lining. This is attributed to the increase in the reflectivity of the furnace, which, in turn, leads to an increase in the density of the radiant flux incident on the surface of the steel billet on the source side of the furnace hearth. This circumstance is the reason for the increase in the total power of radiation heating of the billet, which leads to a reduction in the time of its heating.

The output increases with growth in the height of the furnace arch only to a certain limit, after which we have saturation. This is due to the increase in the optical density of the gas layer between the billet and the furnace wall, which leads to an increase in the emittance of the flue gases and accordingly to an increase in the total power of radiation heating of the billet.

The above regularities of the influence of the height of the furnace arch and the emissivity factor of the furnace lining on the rate of heating of a steel billet are in complete agreement with the regularities of transfer in an absorbing medium (one may read more widely on these regularities in [21–23]).

The next step of investigation was in computing the error arising in calculation of the characteristics of radiation transfer with the use of the gray absorption coefficient (6) and in integration of the spectral characteristics $\bar{Q}_{inc} = \int Q_{inc}(\lambda)d\lambda$ (in our case it is sufficient to consider the spectral range 0.6–10 μm). Figure 7 gives the dependences for the integral radiation fluxes $Q_{inc}^{gr}(T_m)$ averaged over the billet surface and calculated with the use of the "gray" absorption coefficient on the surface temperature of the metal. The results show that the difference between the above values may attain 15–20%, which is significant for solution of the internal problem of heating of a steel billet. Analogous statements can be made for the divergence of radiant fluxes (i.e., radiation heat sources) in the furnace volume which determine the temperature field in it:

$$\operatorname{div} Q_{\mathrm{r}}^{\mathrm{gr}}(\mathbf{r}) = \tilde{\chi}(\mathbf{r}) \left(4\sigma_0 T^4(\mathbf{r}) - \int_{4\pi} I(\mathbf{r}, \Omega)\, d\Omega \right), \tag{13}$$

$$\operatorname{div} \overline{Q}_{\mathrm{r}}^{\mathrm{inc}}(\mathbf{r}) = \int_0^{\infty} \tilde{\chi}_\lambda(\mathbf{r}) \left(4\pi B_\lambda(T(\mathbf{r})) - \int_{4\pi} I_\lambda(\mathbf{r}, \Omega)\, d\Omega \right) d\lambda\,. \tag{14}$$

CONCLUSIONS

1. We have investigated the possibility of reducing the dimension of the problem of numerical calculation of radiation fluxes incident on the walls of an annular furnace in its operation and on the surface of the worked metallic billets of cylindrical and rectangular cross sections.

2. It has been shown that the replacement of the three-dimensional model by a two-dimensional one is expedient if the optical density of the medium is $\tau_X \geq 0.5$ and the ratio of optical densities in mutually perpendicular directions is $\delta = \tau_Z/\tau_X \geq 0.2$. The computational error due to the reduction in the dimension of the problem does not exceed 10%, which may be assumed to be acceptable in solving most practical problems.

3. We have considered the influence of the selectivity of radiation of flue gases and refractory surfaces of the furnace on the integral densities of radiation fluxes.

4. It has been shown that allowance for the spectral properties of the furnace medium in explicit form is necessary to attain a high accuracy of calculation of the characteristics of radiation transfer in the annular-furnace volume. Numerical calculations of these characteristics have shown that the output (efficiency) of the furnace decreases with increase in the emissivity factor of its lining.

5. It has been found that the output increases with growth in the furnace-arch height only to a certain limit, after which we have saturation. The qualitative explanations for the above features are given.

NOTATION

$A_i(\lambda)$, $B_i(\lambda)$, and $C_i(\lambda)$, empirical coefficients; $B(T)$ and $B_\lambda(T)$, integral and spectral intensities of black-body radiation at the temperature T respectively; $B_{\max}(T)$, maximum intensity of black-body radiation at the temperature T; d, radius of a billet; H, height of the furnace arch; $I_\lambda(\mathbf{r}, \mathbf{l})$, spectral intensity of radiation at the point $\mathbf{r}$ in the direction $\mathbf{l}$; $I_{0\lambda}(\mathrm{P}, \mathbf{l})$, spectral intensity of the intrinsic radiation or of radiation transmitted from the outside at the point P of the boundary; J, angular mean intensity of radiation emerging from the medium; L_X, L_Y, and L_Z, sides of the domain having the shape of a parallelepiped and inside which the propagation of radiation is calculated; $\mathbf{n}$, external normal to the heat-absorbing surface; N_{g}, number of different gases in the mixture; $p_\lambda(\mathbf{r}, \mathbf{l}, \mathbf{l}')$, indicatrix of scattering of radiation in its interaction with the volume element of the medium; $p_i(\mathbf{r})$, partial pressure of the ith gas in the mixture; $\mathbf{r}$, radius vector; s, distance between metallic billets; T, temperature; t_{h}, duration of the process of heating of a billet from the instant of charging of the metal into the furnace to the fulfillment of the conditions of completion of the process; $Q_{\mathrm{r}}(\mathbf{r})$, radiation-flux density; $Q_{\mathrm{res}}(\mathbf{r})$, density of the resulting-radiation flux; Q_λ, spectral radiation-flux density; $\overline{Q}$, radiation flux integral over the spectrum; $q_{\mathrm{w}}^{\mathrm{res}}(\mathrm{P})$, local density of the resulting radiation flux onto heat-absorbing surfaces at the point P; q_1 and q_2, heat fluxes incident on a billet from above and from below respectively; U, specific output of the furnace; $\delta = \tau_Z/\tau_X$; $\chi_\lambda(\mathbf{r})$ and $\sigma_\lambda(\mathbf{r})$, spectral coefficients of absorption and scattering respectively; $\chi_{\lambda p}$, specific (per atmosphere) spectral coefficient of absorption; ζ, point at the boundary of a two-dimensional computational domain; $\tilde{\chi}$, absorption coefficient of the furnace medium, mean-integral over the spectrum, or gray coefficient; $\overline{\chi}$, volume-mean absorption coefficient of the furnace medium; $\sigma_0 = 5.68 \cdot 10^{-8}$ W/(m$^2 \cdot$K^4), Stefan–Boltzmann constant; ε, emissivity factor; $\varepsilon(Q_{res})$ and $\varepsilon(J)$, errors of calculation of the flux and mean intensity of radiation, which are determined by the reduction in the dimension of the problem; μ, coefficient of nonuniformity of heating; λ, electromagnetic-radiation wavelength; τ_X, τ_Y, and τ_Z, optical thickness of the medium in the computational domain in the direction of the X, Y, and Z axes respectively; $\rho_\lambda(P, \mathbf{l}, \mathbf{l}')$, spectral reflection coefficient of the boundary; ρ_m, density of the billet metal; Ω, solid angle. Subscripts and superscripts: g, gas; gr, radiation characteristics calculated with the use of the absorption coefficient mean-integral over the spectrum (or the gray coefficient); h, heating; inc, incident; m, metal; max, maximum value; r, radiation; res, resulting; sr, metallic-billet surface; w, wall (boundary of the computational domain); λ, spectral characteristic.

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